Shree H.N. Shukla Group of Colleges

M.Sc. SEMESTER 2

Sub. Code: CMT-2001

Core Sub. 1: Abstract Algebra 2

Question Bank

1) If a field E is a finite extension field of a field F, then E is an algebraic extension of F.

2) Prove that every finite extenstion is an algebraic extension

3) Let E|F and K| E both are algebraic extensions .Prove that K |F is also an algebraic extenstion .

4) Prove that $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \ldots \ldots \sqrt{p}, \ldots) \mid \mathbb{Q}$ is an infinite algebraic extension .

5) Let p(x)∈ F[x] be an irreducible polynomial and deg(p(x)) = n .Let E | F be an extension such that $\alpha \in$ F and α is a root of p(x) .Prove that F[α] = F(α) , [F(α) : F] = n and { 1 , α , α^2, ... α^{n-1} } is a basis of F(α) over F .

6) Let f(x) ∈ F[x] be an irreducible polynomial .Prove that α is a multiple root of f(x) if and only if f ‘ (x) = 0 (All the coefficients of f ‘ (x) are multiple of Char F .)

7) Let char K = p > 0 and f(x) ∈ K[x] be an irreducible polynomial .Prove that f(x) has a multiple root if and only if f(x) = g(x^p) , for some g(x) ∈ K[x] .

8) Let F be a finite field . Prove that $F^* = F \setminus \{0\}$ is a cyclic group under multiplication .

9) Let (Ni) i∈ Λ be a family of R-submodules of an R-module M. Prove that $\bigcap_{i\in\Lambda} N_i$ is also an R- submodule of M .

10) Using Eisenstein criterian prove that g(x) = $1+x+x^2+....+x^{p-1}$ (p is prime) and g(x+1) both are irreducible over ℚ[x] .

11) Let F be a field .Prove that the prime subfield of F is either isomorphic to ℚ or it is isomorphic to ℤp , for some prime p .

12) Let R be a ring with unity . Prove that an R-module M is cyclic iff M ≅ R/ I , for some left ideal I of R .

13) State and prove primitive element theorem .

14) Let f : M N be an R-homomorphism on R- modules . Prove that kerf and f(M) are R-submodules of M and N respectively .

15) Define an exact infinite sequence of R-homomorphisms of R-modules . Suppose following diagram of R-modules and R-homomorphism is commutative and it has exact rows.

$$\begin{tikzcd}
K \arrow[r, "f"'] \arrow[d, "\alpha"] & M \arrow[r, "g"'] \arrow[d, "\beta"] & L \arrow[d, "\gamma"] \\
K' \arrow[r, "f'"'] & M' \arrow[r, "g'"'] & L'
\end{tikzcd}$$

Prove that (i) β is one-one if α, r ,f ' all are one –one maps and (ii) β is onto if α , r , g all are onto maps .