SHREE H. N. SHUKLA GROUP OF COLLEGES

M.Sc. (Mathematics)
Semester-4

PRELIMS TEST

Subject : Integration Theory **Marks : 70**

Date: __ / __ / ____ **Time : 2.5 Hours**

Q-1 Answer any seven questions. **(14)**

1) The lebesuge measure on $\mathbb{R}$ is __________.

2) The cumulative function F of a finite barie measure on the real line is __________.

3) If (X, A, μ) is a complete measure space then $\{s / s$ is simple measurable on X and $\mu\{x \in X / s(x) \neq 0\} < \infty\}$ is dense in __________.

4) Every closed set in a metric set is a __________.

5) Define μ^*-measurable subset of a set x with outer measure μ^*. Prove that every $E \subset X$ with $\mu^* E=0$ is μ^*-measurable.

6) Prove that the product of two σ-finite complete measures is σ-finite.

7) If $f \in L^1(X, \mathcal{A}, \mu)$ then prove that $f(x)$ is finite are $x \in X$.

8) If x is a to topological space and $E \subset X$ then find supp (x_E).

9) Give an example of a space which is locally compact but not compact. Justify.

10) Define σ-finite measure on a measurable space (X, A) and give an example of a σ-finite measure.

Q-2 Answer any two question. **(14)**

1) State and prove Hahn decomposition theorem.

2) Define Jordan decomposition of a signed measure on a measurable space and prove that it is unique.

3) If μ_1, μ_2 are two measures on a measurable space $(X, \mathscr{A})$ and at least one of them is finite then prove that $\mu_1 - \mu_2$ is a signed measure on $(X, \mathscr{A})$.

Q-3 Answer any two questions. **(14)**

1) Define :

   (i) a locally compact and

   (ii) a hausdorff space. Is the set of rationals in $\mathbb{R}$ is locally compact ?

2) Let X is a locally compact hausdroff space. Prove that Ba(X) = the σ–algebra generated by compact G_δ sets in X.

3) Define σ–bdd set in a locally compact hausdorff space X. If $E \in Ba(X)$ then prove that either E or X \ E is σ–bdd.

Q-4 Answer any two questions.

1) If X is a locally compact separable metric space then prove that $Bo(X) = Ba(X)$.

2) Define σ-compact set in a locally compact Hausdorff space. Prove that every σ-compact open set in a locally compact Hausdorff space is a Baire set.

3) Give an example of baire measure on a locally compact Hausdorff space which is not regular. Justify.

Q-5 Answer any two questions.

1) Prove that if (X, A) is a measurable space and $f : X \to [0, \infty]$ be measurable then there exists a sequence $\{S_n\}_{n=1}^{\infty}$ of simple measurable function such that

(i) $0 \leq S_1 \leq S_2 \leq \cdots \ldots\ldots \leq S_n \ldots\ldots \leq f ;\text{on } X.$

(ii) $\lim_{n\to\infty} S_n = f(x); \forall x \in X.$

2) Prove that if X be a countable set and μ be the counting measure then $L^P(\mu) \cong l^P ; \forall\ 1 \leq P < \infty.$

3) Define Baire measure on the real line. Prove that the cumulative distribution function "F" of a finite signed measure on the real line is bdd, monotonically increasing and $\lim_{x\to-\infty} f(x) = 0$.

--------------------------------------- ALL THE BEST ---