Seat No.

Topology - Sem-1
NGP - 2025.
January.

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 1 JANUARY 2025 (Regular) JANUARY - 2025

TOPOLOGY-1

Faculty Code : 003

Subject Code : 16SSMSMA-CO-01-00003

Time : 2:30 Hours] [Total Marks : 70

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 Define topology with example.
2 What is base of discrete topology
3 Define simple order.
4 Define interior of a set.
5 If $A = [0,1) \cup (2,3]$ then find $\bar{A}$.
6 Define continuous function with example.
7 Define homeomorphism. Are (a,b) and (0,1) homeomorphic?
8 Define convergent sequence in topology.
9 Is Q is connected? If no then find the separation of Q.
10 Write any two properties of connected component.

Q.2 Answer the following (Any Two) 14

1 Let X be a topological space and $\mathfrak{B}$ be a basis for the topology on X. Let Y be a subspace of X.

Define $\mathfrak{B}_Y = \{B \cap Y | B \in \mathfrak{B}\}$. Prove that $\mathfrak{B}_Y$ is basis for the subspace topology.

2 Let (X,d) be metric space. Prove that $\mathfrak{B}_d = \{B_d(x,r) : x \in X, r > 0\}$ is a basis for some topology on X.

3 Let $X \neq \phi$ and $\mathfrak{B}$ be a family of collection of subsets of X then $\mathfrak{B}$ is a basis for some topology τ on X iff

i. $\cup\{B : B \in \mathfrak{B}\} = X$
ii. If $B_1, B_2 \in \mathfrak{B}$ and $x \in B_1 \cap B_2$ then there exist $B_3 \in \mathfrak{B}$ such that $x \in B_3 \subset B_1 \cap B_2$.

Q.3 Answer the following 14

1 Let X and Y be spaces and $A \subset X, B \subset Y$ then

i. $\overline{A \times B} = \bar{A} \times \bar{B}$
ii. $(A \times B)^o = A^o \times B^o$

2 Let Y_α is connected subspace of X, for every $\alpha \in I$. Suppose $\cap_{\alpha \in I} Y_\alpha \neq \phi$ then $\cup_{\alpha \in I} Y_\alpha$ is connected subspace of X.

OR

Answer the following 14

Let X, Y and Z be spaces and $f : Z \to X \times Y$. Then f is continuous iff $\pi_x o f : Z \to X$ and $\pi_y o f : Z \to Y$ are continuous.

Let X be a topological space and Y be a subspace of X. Let A be a subset of Y and $\bar{A}$ denote the closure of A in X. Prove that $Cl_Y A = \bar{A} \cap Y$.

Q.4 Answer the following questions (Any Two) 14

1 State and prove pasting lemma.

Let A be subset of a topological space X. Prove that $x \in \bar{A}$ if and only if for every open set U containing x, $U \cap A = \phi$.

Q.5 Answer the following (Any Two) 14

1 Let Z be a subspace of Y and $f : X \to Z$ is a map. Then $f : X \to Z$ is continuous iff $f : X \to Y$ is continuous.

Let X and Y be spaces and $f : X \to Y$. The following statements are equivalents:

i. f is continuous on X
ii. For each closed set K of Y, $f^{-1}(K)$ is closed subset of X.

Let (X,d) and (Y,ρ) be metric spaces. Let $f : X \to Y$ be a function and $a \in X$. Then the following statements are equivalents:

i. $f : (X, \tau(d)) \to (Y, \tau(\rho))$ is continuous.
ii. For every $\epsilon > 0$, there is some $\delta = \delta(x,a)$ such that whenever, $d(x,a) < \delta, \rho(f(x), f(a)) < \epsilon$

4 If X and Y are connected spaces then $X \times Y$ is conncected.

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