Seat No. ________

MASTER OF SCIENCE MATHEMATICS Examination

MSC MATHS Semester - 1 JANUARY 2025 (Regular) JANUARY - 2025

ALGEBRA-1

Faculty Code : 003

Subject Code : 16SI-MSMA-CO-01-00001

Time : 2:30 Hours]

[Total Marks : 70

Instructions: All questions are compulsary

Q.1 Answer Briefly any seven of the following (Out of ten) 14

1 Define a simple group and give an example of a simple group.

2 For a group G, prove that, the conjugate class $C(e)$ generated by the identity e is the single-ton set $\{e\}$.

3 In standard notation, prove or disprove that, S_3 is an abelian group.

4 Write down $\sigma \in S_9$ as a finite product of disjoint cycles, where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 3 & 9 & 5 & 8 & 1 & 7 & 6 & 2 \end{pmatrix}$.

5 Define term: Integral Domain.

6 Define conjugate relation on a group G. Is it an equivalence relation on G?

7 Let $H < G$ and $[G : H] = 2$. Prove that, H is a maximal normal subgroup of G.

8 Define the terms: Commutator element and commutator subgroup.

9 For a ring R, prove that, {0} and R both are ideals of R.

10 For a ring R and its ideal I on R, prove that, if $1 \in I$, then $I = R$.

Q.2 Answer the following (Any Two) 14

1 State and Prove, First Isomorphism Theorem of Rings.

2 State and Prove, Third Isomorphism Theorem of Groups.

3 Let G_1, G_2 be two groups, N_1 be a normal subgroup of G_1 and N_2 be a normal subgroup of G_2. In standard notation prove that, (i) $N_1 \times N_2$ is a normal subgroup of $G_1 \times G_2$ and (ii) $G_1 \times G_2 / N_1 \times N_2 \simeq \left[G_1/N_1\right] \times \left[G_2/N_2\right]$.

Q.3 Answer the following 14

Let G be a finite cyclic group and $o(G) = n \geq 2$. Prove that, Aut (G) = $\{\phi_r : G \to G$ defined by $\phi_r(x) = x^r$, for all $x \in G$ and $1 \leq r \leq n$ with $(n, r) = 1\}$.

2 Let G be a finite group and p divide to the order of the group G. Let n_p be the number of Sylow p-subgroups of G. Prove that, n_p will divide to the order of the group G.

OR

Answer the following 14

In standard notation prove that, $M_n(I)$ is an ideal of $M_n(R)$, where R is a ring and I is its ideal.

2 Let R be a ring and J be an ideal of $M_n(R)$. Prove that, $J = M_n(I)$, for some ideal I of R.

Q.4 Answer the following questions (Any Two) 14

For a group G, in standard notation prove that,

i. G' is normal subgroup of G.

ii. G/G' is an abelian group.

2 For any normal subgroup H of G, if G/H is abelian, then prove that, G' is a subset of H.

14

Answer the following (Any Two)

1

Let R be a ring and A, B be two ideals of R. Prove that, $\{\sum_{i=1}^{t} a_i b_i \ / \ t \geq 1,$ $a_i \in A,\ b_i \in B,$ for all i = 1, 2, 3, … …, t$\}$ and $A \cap B$ both are ideals of R.

Prove that, the collection of all normal subgroups of S_n $(n \geq 5)$ is precisely { {e}, A_n, S_n }.

Let R be a ring and $1 \in R$. Let M be an ideal of R with $M \neq R$. Prove that, M is a maximal ideal of R if and only if R/M is a field.

Let H, K be two distinct maximal normal subgroups of a group G. Prove that, $H \cap K$ is also a maximal normal subgroup of H.